3/12/2020

Hints for Midterm Questions
(Due date: 3/26/2020) (by email)

1) $E[B] = ?$ $\quad B = .5(400 - E)_+$

2) $\mathrm{Var}(X) = \underbrace{E[X^2] - (E[X])^2}_{\text{mix}}$

3) (a,b,0) - distribution (which one?)

4) $X \sim 2\text{-Pareto}(\alpha, \theta) \Rightarrow (X - d \mid X > d) \sim 2\text{-Pareto}(\alpha, \theta + d)$

$E[X - 25 \mid X > 50] = E[X - 50 + 25 \mid X > 50]$ $\quad E[25 \mid X > 50] = 25$

$= E[X - 50 \mid X > 50] + 25$

5)

N	Pr
0	p_0
1	p_1
2	p_2
3	p_3
⋮	⋮
10	p_{10}

N^T	Pr
0	0
1	p_1^T
2	$p_2^T \to ?$
3	p_3^T
⋮	⋮
10	p_{10}^T

(See P. 14;
last sentence)

6) $X \sim Exp(\theta = 1000)$ $\quad d = 100$

? $Var(Y^L)$? $\quad Y^L = (X - 100)_+$

Remiders: $Y^P = X - d \mid X > d \sim Exp(\theta = 1000)$

$$E[(Y^L)^k] = E[(Y^P)^k] \cdot Pr(X > d)$$

7) $Y^P = X \mid X > d$ when d = Franchise deductible

? $E[X \mid X > 300] = E[X - 300 + 300 \mid X > 300]$

8) $N \mid \Lambda \sim P(\Lambda)$ & $\Lambda \sim \Gamma(\alpha, \theta)$

$$\Longrightarrow N \sim NB(r = \alpha, \beta = \theta)$$

9) $X_{2020} \sim Exp(\theta = 10)$

$X_{2021} = 1.1 \cdot X_{2020} \sim Exp(\theta = 11)$

$Y^P_{2021} = X_{2021} - 2 \mid X_{2021} > 2 \sim Exp(\theta = 11)$

10) $A = Pr(\overset{\text{convoluted event}}{Max(X_1, X_2, X_3) > 400})$

$= 1 - Pr(max(X_1, X_2, X_3) \leq 400)$

$= 1 - [Pr(X \leq 400)]^3$

11) ?$E[Y^L]$ $\qquad Y^L = \alpha(X - d)_+$

12) $$Y^L = \begin{cases} 0 & \text{if } X < 200 \\ X & \text{if } 200 < X < 2000 \\ 2000 & \text{if } X > 2000 \end{cases}$$

$$E[Y^L] = E[Y^P] \cdot Pr(X > d)$$

13) $Y = X - 4 \mid X > 4 < 8 \iff X < 12 \mid X > 4$

$$Pr(Y < 8) = Pr(X < 12 \mid X > 4) = \frac{Pr(4 < X < 12)}{Pr(X > 4)}$$

14) $M = N_1 + N_2 + \cdots + N_5 \sim NB(r = 5, \beta) \qquad \beta = 2$

? $Pr(M = 0 \text{ or } 1)$

15) $T \mid Y \sim Exp\left(mean = \frac{1}{Y}\right)$ & $Y \sim \Gamma(\alpha, \theta)$

$$\Longrightarrow T \sim \text{2-Pareto}(\tilde{\alpha}, \tilde{\theta}) \qquad \tilde{\alpha} = \alpha$$

$$\tilde{\theta} = \frac{1}{\theta}$$

16) $\cancel{X}$ $X \mid \theta \sim N(\mu = \theta, \sigma^2 = 8000)$ & $\theta \sim N(\mu = 1000, \sigma^2 = A)$

$$\Longrightarrow X \sim N(\mu = 8000, \sigma^2 = 1000 + A)$$

17) $P_K^M = \dfrac{1 - P_0^M}{1 - P_0} \cdot P_K$ $\qquad \dfrac{P_K^M}{P_{K-1}^M} = \dfrac{P_K}{P_{K-1}} = a + \dfrac{b}{K}$

18) $N \mid \Lambda \sim P(\Lambda)$ $\quad \Lambda \sim U(0, 10)$

? $Var(N)$ (Use Law of Total Variance)

19) ✓

20) $N = N_1 + N_2 + N_3 \sim B(m = 6, q)$

↳ $P_2 = ?$